

《高校数学公式集》

(不等式)

1. $\frac{a+b}{2} \geq \sqrt{ab}$, $\frac{a+b+c}{3} \geq \sqrt[3]{abc}$ (相加・相乗平均, a, b, c は正または0)
2. $(a^2+b^2+c^2)(x^2+y^2+z^2) \geq (ax+by+cz)^2$ (コーシー・シュワルツの不等式)

(三角比)

3. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ (正弦定理)
4. $a^2 = b^2 + c^2 - 2bc \cos A$ (余弦定理)
5. $S = \frac{1}{2}bc \sin A = \sqrt{s(s-a)(s-b)(s-c)}$: (ヘロンの公式) ($s = \frac{1}{2}(a+b+c)$)

(図形と式)

6. 数直線上の2点 x_1, x_2 を $m:n$ に内分および外分する点:

$$\frac{nx_1+mx_2}{m+n}, \frac{-nx_1+mx_2}{m-n}$$

7. 点 (x_1, y_1) と直線 $ax+by+c=0$ との距離: $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

点 (x_1, y_1, z_1) と平面 $ax+by+cz+d=0$ との距離: $\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$

8. だ円 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 上の点 (x_1, y_1) における接線: $\frac{x_1x}{a^2} + \frac{y_1y}{b^2} = 1$
9. 双曲線 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 上の点 (x_1, y_1) における接線: $\frac{x_1x}{a^2} - \frac{y_1y}{b^2} = 1$

(ベクトルと行列)

10. 内積 $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$
11. $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ ($ad-bc \neq 0$)
12. $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ のとき, $A^2 - (a+d)A + (ad-bc)E = O$ E は単位行列, O は零行列

【参考】 点 (x, y) を原点の周りに角 θ だけ回転した点を (x', y') とすれば,

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \iff x' + y'i = (\cos \theta + i \sin \theta)(x + yi)$$

(複素数)

13. 極形式: $z = r(\cos \theta + i \sin \theta)$ ($r = |z|$, $\theta = \arg z$)
14. $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$, $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$ に対し,
 $z_1 z_2 = r_1 r_2 \{\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)\}$

15. ド・モアブルの定理: $z = r(\cos \theta + i \sin \theta)$ に対し, $z^n = r^n(\cos n\theta + i \sin n\theta)$
(解と係数の関係)

16. $x^2 + px + q = 0$ の解が α, β のとき, $\alpha + \beta = -p$, $\alpha\beta = q$
17. $x^3 + px^2 + qx + r = 0$ の解が α, β, γ のとき,
 $\alpha + \beta + \gamma = -p$, $\alpha\beta + \beta\gamma + \gamma\alpha = q$, $\alpha\beta\gamma = -r$

(対数)

18. ① $\log_a MN = \log_a M + \log_a N$ ② $\log_a \frac{M}{N} = \log_a M - \log_a N$
③ $\log_a M^p = p \log_a M$ ④ $\log_a M = \frac{\log_b M}{\log_b a}$

(三角関数)

19. $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$ (加法定理)
 $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
20. $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$
21. $\sin 2\alpha = 2 \sin \alpha \cos \alpha$ $\cos 2\alpha = 1 - 2 \sin^2 \alpha = 2 \cos^2 \alpha - 1$ (2倍角の公式)

22. $\sin \alpha \cos \beta = \frac{1}{2} \{\sin(\alpha + \beta) + \sin(\alpha - \beta)\}$ (積→和の変換公式)
 $\cos \alpha \sin \beta = \frac{1}{2} \{\sin(\alpha + \beta) - \sin(\alpha - \beta)\}$
 $\cos \alpha \cos \beta = \frac{1}{2} \{\cos(\alpha + \beta) + \cos(\alpha - \beta)\}$
 $\sin \alpha \sin \beta = -\frac{1}{2} \{\cos(\alpha + \beta) - \cos(\alpha - \beta)\}$

23. $\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$ (和→積の変換公式)
 $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$
 $\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$
 $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$

24. $a \sin \theta + b \cos \theta = \sqrt{a^2 + b^2} \sin(\theta + \alpha)$ $\sin \alpha = \frac{b}{\sqrt{a^2 + b^2}}$, $\cos \alpha = \frac{a}{\sqrt{a^2 + b^2}}$

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(数列)

$$25. S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\} \quad (l=a+(n-1)d)$$

$$26. \text{初項 } a, \text{ 公比 } r, \text{ 項数 } n \text{ の等比数列の和: } S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$27. 1+2+3+\cdots+n = \frac{1}{2}n(n+1) \quad 1^2+2^2+3^2+\cdots+n^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$1^3+2^3+3^3+\cdots+n^3 = \left\{ \frac{1}{2}n(n+1) \right\}^2$$

(極限)

$$28. \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \lim_{h \rightarrow 0} (1+h)^{\frac{1}{h}} = e = 2.71828\cdots$$

$$29. \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

(微分)

$$30. \{f(g(x))\}' = f'(g(x))g'(x)$$

$$31. x=f(y) \text{ のとき, } \frac{dy}{dx} = \left(\frac{dx}{dy}\right)^{-1}$$

$$32. x=x(t), y=y(t) \text{ のとき, } \frac{dy}{dx} = \frac{y'(t)}{x'(t)}$$

$$33. (\tan x)' = \frac{1}{\cos^2 x}, (\log x)' = \frac{1}{x}$$

(積分)

$$34. \int f(g(t))g'(t)dt = \int f(x)dx$$

$$35. \int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$$

$$36. \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$$

$$37. \int \log x dx = x \log x - x + C$$

$$38. \int_0^a \sqrt{a^2-x^2} dx = \frac{1}{4}\pi a^2, \int_0^a \frac{1}{x^2+a^2} dx = \frac{\pi}{4a},$$

$$\int_\alpha^\beta (x-\alpha)(x-\beta)dx = -\frac{1}{6}(\beta-\alpha)^3$$

$$39. \text{回転体の体積: } V = \pi \int_a^b \{f(x)\}^2 dx$$

$$40. \text{曲線の長さ: } \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

(順列・組合せ)

$$41. {}_n P_r = \frac{n!}{(n-r)!}, {}_n C_r = \frac{{}_n P_r}{r!} = \frac{n!}{r!(n-r)!}$$

$${}_n C_r = {}_{n-1} C_r + {}_{n-1} C_{r-1}, r \cdot {}_n C_r = n \cdot {}_{n-1} C_{r-1} \quad (1 \leq r \leq n-1)$$

$$42. (a+b)^n = \sum_{r=0}^n {}_n C_r a^{n-r} b^r: \text{二項定理}$$

(確率)

$$43. \text{確率 } p \text{ の事象が } n \text{ 回の試行中 } r \text{ 回起こる確率: } P = {}_n C_r p^r (1-p)^{n-r}$$

$$44. \text{確率変数 } X \text{ の平均(期待値): } E(X) = \sum_{k=1}^n x_k p_k = m$$

$$\text{分散: } V(X) = \sum_{k=1}^n (x_k - m)^2 p_k = \sum_{k=1}^n x_k^2 p_k - m^2$$

$$\text{標準偏差: } \sigma(X) = \sqrt{V(X)}$$

$$45. \text{二項分布 } B(n, p) \text{ については, } E(X) = np, V(X) = np(1-p)$$

(極座標)

46. 媒介変数表示

$$\text{円: } x^2 + y^2 = r^2 \quad \Leftrightarrow \quad x = r \cos \theta, y = r \sin \theta$$

$$\text{だ円: } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \Leftrightarrow \quad x = a \cos \theta, y = b \sin \theta$$

$$\text{双曲線: } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \Leftrightarrow \quad x = \frac{a}{\cos \theta}, y = b \tan \theta$$

$$\text{放物線: } y^2 = 4px \quad \Leftrightarrow \quad x = pt^2, y = 2pt$$

$$47. \text{直交座標 } (x, y) \text{ を極座標 } (r, \theta) \text{ で表すと, } x = r \cos \theta, y = r \sin \theta$$

$$r = \sqrt{x^2 + y^2}, \theta \text{ は始線とのなす角(偏角).}$$

48. 2点 A(r_1, θ_1), B(r_2, θ_2) について,

$$AB = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_1 - \theta_2)}, \triangle OAB = \frac{1}{2} r_1 r_2 |\sin(\theta_1 - \theta_2)|$$