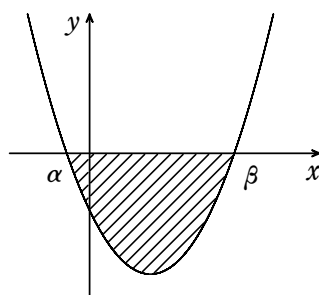


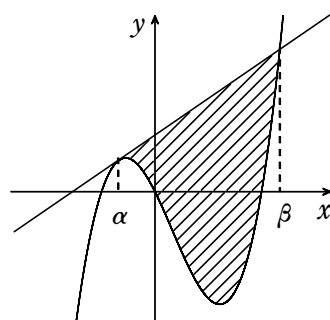
$$\ast \int_{\alpha}^{\beta} (x-\alpha)(x-\beta) dx = -\frac{1}{6}(\beta-\alpha)^3 : \langle \text{面積公式①} \rangle$$

1 ◎ <種々の面積公式>

$$\begin{aligned} \textcircled{1} \int_{\alpha}^{\beta} (x-\alpha)(x-\beta) dx &= \int_{\alpha}^{\beta} (x-\alpha)\{x-\alpha-(\beta-\alpha)\} dx \\ &= \int_{\alpha}^{\beta} \{(x-\alpha)^2 - (\beta-\alpha)(x-\alpha)\} dx \\ &= \left[ \frac{(x-\alpha)^3}{3} - \frac{(\beta-\alpha)}{2}(x-\alpha)^2 \right]_{\alpha}^{\beta} \\ &= \frac{1}{3}(\beta-\alpha)^3 - \frac{1}{2}(\beta-\alpha)^3 \\ &= -\frac{1}{6}(\beta-\alpha)^3 \end{aligned}$$



$$\begin{aligned} \textcircled{2} \int_{\alpha}^{\beta} (x-\alpha)^2(x-\beta) dx &= \int_{\alpha}^{\beta} (x-\alpha)^2\{x-\alpha-(\beta-\alpha)\} dx \\ &= \int_{\alpha}^{\beta} \{(x-\alpha)^3 - (\beta-\alpha)(x-\alpha)^2\} dx \\ &= \left[ \frac{(x-\alpha)^4}{4} - \frac{(\beta-\alpha)}{3}(x-\alpha)^3 \right]_{\alpha}^{\beta} \\ &= \frac{1}{4}(\beta-\alpha)^4 - \frac{1}{3}(\beta-\alpha)^4 = -\frac{1}{12}(\beta-\alpha)^4 \\ &= -\frac{1}{12}(\beta-\alpha)^4 \end{aligned}$$



$$\begin{aligned} \textcircled{3} \int_{\alpha}^{\beta} (x-\alpha)^2(x-\beta)^2 dx &= \int_{\alpha}^{\beta} (x-\alpha)^2\{(x-\alpha)^2 - 2(\beta-\alpha)(x-\alpha) + (\beta-\alpha)^2\} dx \\ &= \int_{\alpha}^{\beta} \{(x-\alpha)^4 - 2(\beta-\alpha)(x-\alpha)^3 + (\beta-\alpha)^2(x-\alpha)^2\} dx \\ &= \left[ \frac{(x-\alpha)^5}{5} - \frac{(\beta-\alpha)}{2}(x-\alpha)^4 + \frac{1}{3}(\beta-\alpha)^2(x-\alpha)^3 \right]_{\alpha}^{\beta} \\ &= \frac{1}{5}(\beta-\alpha)^5 - \frac{1}{2}(\beta-\alpha)^5 + \frac{1}{3}(\beta-\alpha)^5 \\ &= \frac{1}{30}(\beta-\alpha)^5 \end{aligned}$$

