※ $\boxed{\displaystyle\int_{\alpha}^{\beta}(x-\alpha)(x-\beta)dx=-\frac{1}{6}(\beta-\alpha)^3}$ ：《面積公式①》

$\boxed{1}$ ◎ ＜種々の面積公式＞

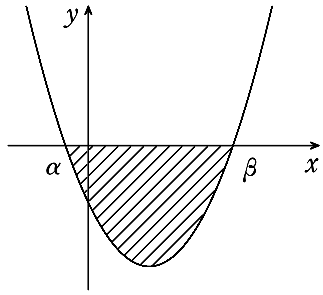
① $\displaystyle\int_{\alpha}^{\beta}(x-\alpha)(x-\beta)dx=\int_{\alpha}^{\beta}(x-\alpha)\{x-\alpha-(\beta-\alpha)\}dx$

$\displaystyle=\int_{\alpha}^{\beta}\{(x-\alpha)^2-(\beta-\alpha)(x-\alpha)\}dx$

$\displaystyle=\left[\frac{(x-\alpha)^3}{3}-\frac{(\beta-\alpha)}{2}(x-\alpha)^2\right]_{\alpha}^{\beta}$

$\displaystyle=\frac{1}{3}(\beta-\alpha)^3-\frac{1}{2}(\beta-\alpha)^3$

$\displaystyle=-\frac{1}{6}(\beta-\alpha)^3$



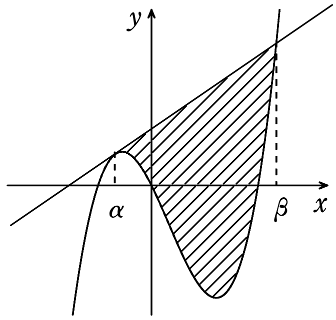
② $\displaystyle\int_{\alpha}^{\beta}(x-\alpha)^2(x-\beta)dx=\int_{\alpha}^{\beta}(x-\alpha)^2\{x-\alpha-(\beta-\alpha)\}dx$

$\displaystyle=\int_{\alpha}^{\beta}\{(x-\alpha)^3-(\beta-\alpha)(x-\alpha)^2\}dx$

$\displaystyle=\left[\frac{(x-\alpha)^4}{4}-\frac{(\beta-\alpha)}{3}(x-\alpha)^3\right]_{\alpha}^{\beta}$

$\displaystyle=\frac{1}{4}(\beta-\alpha)^4-\frac{1}{3}(\beta-\alpha)^4=-\frac{1}{12}(\beta-\alpha)^4$

$\displaystyle=-\frac{1}{12}(\beta-\alpha)^4$



③ $\displaystyle\int_{\alpha}^{\beta}(x-\alpha)^2(x-\beta)^2dx=\int_{\alpha}^{\beta}(x-\alpha)^2\{(x-\alpha)^2-2(\beta-\alpha)(x-\alpha)+(\beta-\alpha)^2\}dx$

$\displaystyle=\int_{\alpha}^{\beta}\{(x-\alpha)^4-2(\beta-\alpha)(x-\alpha)^3+(\beta-\alpha)^2(x-\alpha)^2\}dx$

$\displaystyle=\left[\frac{(x-\alpha)^5}{5}-\frac{(\beta-\alpha)}{2}(x-\alpha)^4+\frac{1}{3}(\beta-\alpha)^2(x-\alpha)^3\right]_{\alpha}^{\beta}$

$\displaystyle=\frac{1}{5}(\beta-\alpha)^5-\frac{1}{2}(\beta-\alpha)^5+\frac{1}{3}(\beta-\alpha)^5$

$\displaystyle=\frac{1}{30}(\beta-\alpha)^5$